# Written Exam at the Department of Economics summer 2021

## **Distributive Justice**

**Final Exam** 

June 2, 2021

(3-hour closed book exam)

Answers only in English.

## This exam question consists of 3 pages in total

#### Falling ill during the exam

If you fall ill during an examination at Peter Bangsvej, you must:

- submit a blank exam paper.
- leave the examination.
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five

(5) days from the date of the exam.

## Be careful not to cheat at exams!

You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

Problem 1 (25%)

Utilitarian welfare is measured by the sum of the utility achieved by each individual in society.

(a) When we study consumer theory, we generally assume that individuals have complete, transitive and continuous preferences defined over the commodity space. Suppose the commodity space is onedimensional (e.g., income). A utility function  $u: \mathbb{R}_+ \to \mathbb{R}$  represents preferences  $\gtrsim$  if and only if for each pair  $x, x' \in \mathbb{R}_+$ ,

$$u(x) \ge u(x') \Leftrightarrow x \gtrsim x'.$$

Now, instead of *u* take a function v = f(u). Argue based on this new function that there exists infinite utility functions representing the same preference.

(b) Above the only information needed from utilities is ordinal. What happens to the utilitarian optimum if we use different utility functions? Elaborate assuming that there are only two individuals, i and j. Then Utilitarianism measures social welfare by:

$$W(x_i, x_j) = u_i(x_i) + u_j(x_j).$$

(c) Utilitarianism builds on interpersonally comparable and cardinal information about utilities. How can this be expressed in terms of  $v_i = f_i(u_i)$  and  $v_j = f_j(u_j)$  in  $W(x_i, x_j)$ ?

(d) Which problems might we run into if we do value judgements with ordinal information?

#### Problem 2 (25%)

There are different types of ethical conditions for intergenerational preferences.

(a) Which axioms related to equal treatment and sensitivity does Maximin satisfy? Argue using the following two streams. The streams record the wellbeing level for each generation.

Stream 1: (1 + 1/1, 1 + 1/2, 1 + 1/3, 1 + 1/4, 1 + 1/5, 1 + 1/6, ...). Stream 2: (1, 1, 1, 1, 1, 1, ...).

(b) Which axioms related to equal treatment and sensitivity does Time-discounted Utilitarianism satisfy? Argue using the following two streams.

Stream 1: (3, 1, 3, 3, 3, 3, ...). Stream 2: (1, 3, 3, 3, 3, 3, ...).

(c) It is generally difficult to jointly satisfy axioms related to equal treatment and sensitivity, but the axioms of Strong Pareto and Finite Anonymity are not in conflict by themselves. Give an example of an additional condition that ensures that both Strong Pareto and Finite Anonymity are satisfied.

#### Problem 3 (25%)

(a) Let F be a cumulative distribution function with mean  $\mu$ . Write the inverse of F as  $F^{-1}$ . The Lorenz curve of F is defined by

$$L(u) = \frac{\int_0^u F^{-1}(t)dt}{\mu}, 0 \le u \le 1.$$

Interpret the Lorenz curve.

(b) Which basic axioms does the Lorenz curve satisfy and how?

(c) Aaberge (2007) introduces the following transformation of the Lorenz curve

$$M(u) = \begin{cases} \frac{L(u)}{u}, & 0 < u \le 1, \\ 0, & u = 0 \end{cases}$$

and names it the normalized Lorenz curve. Which attractive properties does it satisfy?

(d) A popular approach when choosing measures of inequality is to combine the Gini coefficient with measures of the primal family. Why might this be problematic?

(e) Why might Gini's nuclear family be attractive alternative measures of inequality?

## Problem 4 (25%)

Atkinson, Piketty and Saez (2011) describe the evolution of top incomes.

(a) Why should we care about top incomes? Discuss.

(b) Over the last 30-40 years top income shares have increased in some countries but not in others. Discuss theories of what might explain these patterns.

(c) What are some of the limitations of using the tax definition of income when studying top incomes?